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Effective Lagrangian for scale-invariant quantum electrodynamics

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In the critical scaling region of quenched, planar QED a composite scalar state plays an essential role in the effective dynamics. We construct an effective potential describing the dynamics of this state for both weak and strong gauge coupling. The scalar propagator at the strong-coupling end point is also constructed using dispersion relation techniques. The role of the four-fermion interaction in the critical region is emphasized and the fate of the scale symmetry elucidated.

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I. INTRODUCTION

Approximate scale invariance plays a significant role in many diverse areas of theoretical investigation ranging from condensed-matter physics to models of dynamical electroweak symmetry breaking. Quenched, planar (ladder) QED has a vanishing perturbative scale anomaly while exhibiting a dynamical structure [1–7] which includes a nontrivial chiral-symmetry phase structure. It thus provides an attractive arena in which to study the interplay of scale symmetry and dynamical chiral-symmetry breaking and serves as a useful laboratory for more complicated gauge theories with slowly running couplings such as walking technicolor theories [8]. The original motivation [9] which led to our consideration of this model stemmed from the observation that dynamical chiral-symmetry breaking might trigger a spontaneous breakdown of the scale symmetry provided that, at the chiral-symmetry-breaking scale, the scale anomaly is but a small effect. It was speculated that, under such circumstances, the spectrum might contain an abnormally light scalar excitation corresponding to the pseudo Nambu-Goldstone boson of spontaneous scale-symmetry breaking—the dilaton. However, no such nearly massless scalar state appeared for ladder QED. In this paper, we shall clarify the fate of the scale symmetry as well as focus on the dynamical properties of the composite scalar state in this model.

In our previous work [6–7, 10] we found that chirally invariant four-fermion operators play an important role in the dynamical structure of quenched, planar QED. As such we are led to study a gauged U(1) chirally invariant Nambu–Jona-Lasinio (NJL) model in ladder approximation described by the Lagrangian

$$\mathcal{L}=i\bar{\psi}\gamma\cdot D\psi-\mu_0\bar{\psi}\psi+\frac{G_0}{2}[(\bar{\psi}\psi)^2+(\bar{\psi}i\gamma_5\psi)^2]\ , \quad (1.1)$$

where μ_0 provides a soft explicit chiral- and scale-symmetry breaking. In Landau gauge, the fermion self-energy $\Sigma(p)$ is the nonperturbative solution to the ladder approximated Schwinger-Dyson equation with an ultraviolet asymptotic solution given by

$$\Sigma(p)\sim\begin{cases}\tilde{A}\dfrac{\Sigma_0^2}{p}\dfrac{\sinh[\omega\ln(pe^{\delta}/\Sigma_0)]}{\omega}, & \alpha<\alpha_c=\dfrac{\pi}{3}\ , \quad (1.2a)\\[2ex] \tilde{A}\dfrac{\Sigma_0^2}{p}\dfrac{\sin[\tilde{\omega}\ln(pe^{\delta}/\Sigma_0)]}{\tilde{\omega}}, & \alpha>\alpha_c\ , \quad (1.2b)\end{cases}$$

where $\omega=\sqrt{1-\alpha/\alpha_c}$ and $\tilde{\omega}=\sqrt{\alpha/\alpha_c-1}$. Here $\tilde{A}=\tilde{A}(\alpha)$ and $\delta=\delta(\alpha)$ are parameters of the solution while $\Sigma_0=\Sigma(0)$ is the dynamically generated fermion mass scale. Note that the weak gauge coupling ($\alpha<\alpha_c$) solution is power-law behaved, the strong-coupling ($\alpha>\alpha_c$) solution is characterized by an oscillatory factor, and the critical solution ($\alpha\to\alpha_c$) is log behaved. The fermion bare mass

$$m_0=\mu_0-G_0\langle\bar{\psi}\psi\rangle \quad (1.3)$$

enters the solution as an ultraviolet boundary condition

$$m_0=\Sigma(\Lambda)+\frac{\Lambda}{2}\partial_p\Sigma(p)|_{p=\Lambda}\ , \quad (1.4)$$

while the fermion condensate $\langle\bar{\psi}\psi\rangle$ is an order parameter for dynamical chiral-symmetry breaking which is evaluated using the asymptotic solution as

$$\langle\bar{\psi}\psi\rangle=\frac{\Lambda^3}{2\pi^2}\frac{\alpha_c}{\alpha}\partial_p\Sigma(p)|_{p=\Lambda}\ . \quad (1.5)$$

Combining the expressions for m_0 and $\langle\bar{\psi}\psi\rangle$ in Eq. (1.3) yields the gap equation for Σ_0 [6,7].

With the inclusion of the four-fermion interactions, the gap equation exhibits nontrivial chiral-symmetry-breaking solutions for Σ_0 in the chiral limit ($\mu_0=0$) for all values of α. In fact there is a critical curve in the (G,α) coupling-constant plane as shown in Fig. 1, above which the chiral symmetry is dynamically broken and along which a nontrivial continuum limit ($\Lambda/\Sigma\to\infty$) ap-

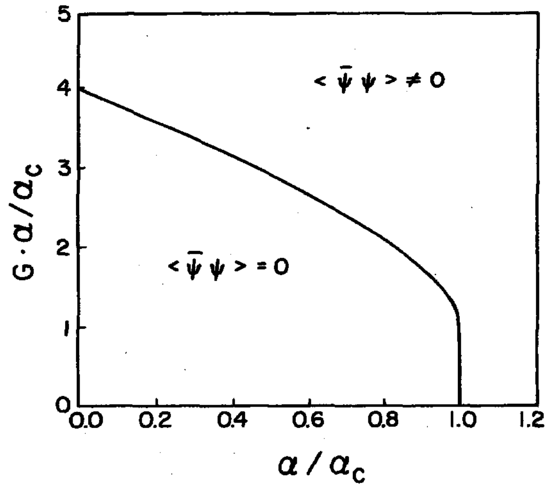

FIG. 1. Chiral symmetry phase diagram.

pears to exist. The critical curve for $0<\alpha<\alpha_c$ follows from the gap equation in the chiral limit and is computed [12] as $G\alpha/\alpha_c=(1+\omega)^2$, where the combination $G=(G_0\Lambda^2/\pi^2)\alpha_c/\alpha$ has been introduced. One end point is at $\alpha=0$ and corresponds to the ordinary NJL model [13], while the other end point occurs at $\alpha=\alpha_c$ and $G\rightarrow 1$ where the solution [6] [Eq. (1.2)] is log behaved.

Composite states are reflected by the appearance of poles in the fermion-antifermion scattering amplitude or equivalently as zeros in the renormalized denominator functions [6]. For the pseudoscalar channel, $D_P^R(0)$ vanishes with the gap in the chiral limit for all α, thus signaling the emergence of the massless Nambu-Goldstone pseudoscalar of spontaneous chiral-symmetry breaking. This is to be contrasted with the scalar denominator function, which after application of the gap equation in the chiral limit, is given by

$$D_S^R(0)=\frac{\tilde{A}^2}{4\pi^2}\Sigma_0^2\left[\frac{2\alpha_c}{\alpha}+1+G\right]. \tag{1.6}$$

As such, no massless scalar (no dilaton) emerges and the scale invariance appears to be explicitly broken when the nontrivial fermion mass scale Σ_0 is generated.

In Sec. II, we employ the model solution to construct an effective potential [10,11] describing the interactions of the composite scalar degree of freedom. From the form of this potential, the nature of the scale symmetry and its interplay with the chiral symmetry breakdown becomes much clearer. In particular, along the chiral line separating the two distinct chiral symmetry phases, the scale symmetry is preserved but in the Wigner-Weyl mode. Off this line, in both the symmetric and spontaneously broken chiral symmetry phase, the scale symmetry is explicitly broken and the source of the breaking is identified.

In Sec. III we examine the strong gauge coupling end point of the critical line and deduce the form of the scalar effective potential. We introduce a dispersion relation for the QED ladder-dressed bubble function from which the fermion-antifermion scattering amplitude is shown to exhibit a massive pole in the scalar channel. The resultant scalar bound-state mass is again found to be of the order of the dynamically generated fermion mass scale. Finally in Sec. IV we offer some conclusions.

II. EFFECTIVE POTENTIAL: $\alpha<\alpha_c$

The description of the model in the vicinity of the critical line requires the inclusion of all the relevant physical degrees of freedom. To incorporate the composite scalar (σ) and pseudoscalar (π) modes, we recast the model Lagrangian as

$$\mathcal{L}=i\bar{\psi}\gamma D\psi-\bar{\psi}(\sigma+i\gamma_5\pi)\psi-\frac{1}{2G_0}[(\sigma-\mu_0)^2+\pi^2]\,. \tag{2.1}$$

Upon application of the σ,π Euler-Lagrangian equations, this reproduces Eq. (1.1). From the form of Lagrangian (2.1) we identify the full fermion bare mass as

$$\langle\sigma\rangle=m_0\,, \tag{2.2}$$

with m_0 given by Eq. (1.4).

On the other hand, to describe the infrared physics, rather than eliminating the σ,π fields, it proves convenient to integrate out the short-distance components of the fermion field. So doing, we construct the effective potential

$$V(\sigma,\pi)=W(\sigma,\pi)+\frac{1}{2G_0}[(\sigma-\mu_0)^2+\pi^2]\,. \tag{2.3}$$

The contribution $W(\sigma,\pi)$ arises from the fermion determinant while the remaining term is due to the four-fermion interaction and the soft explicit chiral-symmetry breaking. The form of W can be extracted from its vacuum value which satisfies

$$\partial_{m_0}W(m_0,0)=\langle\bar{\psi}\psi\rangle\,, \tag{2.4}$$

where $\langle\bar{\psi}\psi\rangle$ includes all the radiative corrections of ladder QED and is given in Eq. (1.5) as a function of Σ_0. When used in conjunction with Eq. (1.4), which gives the m_0 dependence of Σ_0,

$$\Sigma_0=\left[\frac{\tilde{A}}{4}\left[1+\frac{1}{\omega}\right]e^{\omega\delta}\Lambda^{-1+\omega}\right]^{-1/(2-\omega)}m_0^{1/(2-\omega)}+\cdots\,, \tag{2.5}$$

we have a set of parametric equations which can be integrated to give $W(m_0,0)$. To obtain $W(\sigma,\pi)$ we then need merely make the replacement $m_0^2\rightarrow\sigma^2+\pi^2$.

Because of the power-law nature of the solution for the coupling range $0<\alpha<\alpha_c$, the above procedure can be straightforwardly implemented, yielding

$$W(\sigma,\pi)=-\frac{\Lambda^2}{2\pi^2}\frac{1}{(1+\omega)^2}(\sigma^2+\pi^2)$$
$$+\frac{\tilde{A}^2}{16\pi^2}\frac{\alpha_c}{\alpha}\left[\frac{2}{\omega}-1\right]\left[\frac{\tilde{A}}{4}\left[1+\frac{1}{\omega}\right]e^{\omega\delta}\Lambda^{-1+\omega}\right]^{-4/(2-\omega)}(\sigma^2+\pi^2)^{2/(2-\omega)}+\cdots\ , \tag{2.6}$$

Adding the effects of the four-fermion contribution then gives the full effective potential

$$V(\sigma,\pi)=\frac{\mu_0^2}{2G_0}-\frac{\mu_0}{G_0}\sigma+\frac{1}{2}\left[\frac{1}{G_0}-\frac{\Lambda^2}{\pi^2}\frac{1}{(1+\omega)^2}\right](\sigma^2+\pi^2)$$
$$+\frac{\tilde{A}^2}{16\pi^2}\frac{\alpha_c}{\alpha}\left[\frac{2}{\omega}-1\right]\left[\frac{\tilde{A}}{4}\left[1+\frac{1}{\omega}\right]e^{\omega\delta}\Lambda^{-1+\omega}\right]^{-4/(2-\omega)}(\sigma^2+\pi^2)^{2/(2-\omega)}+\cdots\ . \tag{2.7}$$

The first two terms are μ_0 dependent and provide a soft explicit chiral- and scale-symmetry breaking, while the third term is a quadratically divergent effective-mass term. Since the mass operator $\bar{\psi}\psi$ carries physical scaling mass dimension [6] $d_{\bar{\psi}\psi}=2+\omega$ and noting that the Lagrangian piece $\sigma\bar{\psi}\psi$ has mass dimension 4, it follows that the physical mass dimension of the σ field is

$$d_\sigma=2-\omega\equiv 1+\eta/2\ , \tag{2.8}$$

with $\eta/2=1-\omega$ being the anomalous dimension. Consequently the last term in Eq. (2.7) is a scale-invariant potential.

In the chiral limit ($\mu_0=0$), when the four-fermion coupling G_0 is tuned to be along the critical line,

$$G_0=\frac{\Lambda^2}{\pi^2}\frac{1}{(1+\omega)^2}\ ,$$

then the quadratically divergent effective-mass term vanishes and the effective potential is scale invariant. However, since along this curve the fermion mass scale Σ_0 vanishes in the chiral limit, the chiral symmetry remains unbroken and the scale symmetry is realized in the manner of Wigner and Weyl and not as a spontaneously broken symmetry. In order to spontaneously break the chiral symmetry for $\alpha<\alpha_c$, the four-fermion coupling must be larger than its critical value. Once off the critical line, however, the quadratically divergent effective-mass term ceases to vanish and the scale symmetry is explicitly broken. This breaking reflects the dimensionality of the four-fermion coupling G for $\alpha<\alpha_c$. That is, since the fermion mass operator carries dimension $d_{\bar{\psi}\psi}=2+\omega$, it follows that in ladder approximation, the four-fermion interaction has mass dimension $d_{(\bar{\psi}\psi)^2}=2d_{\bar{\psi}\psi}>4$ and is formally irrelevant for $\alpha<\alpha_c$. Nonetheless, it plays an important role in the dynamical structure of ladder QED. Unless the four-fermion coupling is tuned to be near the critical line, the effective-mass term will dominate the potential and the σ,π fields remain static. When the tuning does occur, however, the composite σ,π degrees of freedom begin propagating and the physics in the vicinity of the critical line can be understood in terms of their dynamics. Thus the irrelevant four-fermion interactions play the crucial role of bringing the theory to the critical region. Once in the critical region, they are replaced by the relevant interactions of the composite scalar and pseudoscalar degrees of freedom. This interpretation is consistent with the results from recent lattice simulations for quenched QED [14].

The masses of the composite pseudoscalar and scalar states can be obtained from the second derivatives of the effective potential after a rescaling by the appropriate wave-function renormalization factors [15] for these states. One means of estimating these factors is to use an analogue of the Pagels-Stokar formula [16] for the pseudoscalar decay constant. Introducing an external axial-vector gauge field A_μ, which couples to the axial-vector current $\bar{\psi}\gamma^\mu\gamma_5\psi$, and performing a momentum expansion of the scale-invariant effective action in the broken phase for $p^2\ll m_0^2$ so that only the lowest-order momentum-dependent term need be retained, leads to the covariant kinetic term

$$\frac{Z_H}{2}\left[\frac{m_0^2}{\sigma^2+\pi^2}\right]^{\eta/(2+\eta)}|D_\mu(\sigma+i\pi)|^2$$
$$=\cdots-2Z_H m_0 A^\mu\partial_\mu\pi\cdots\ , \tag{2.9}$$

which exhibits the direct coupling of the axial gauge field to the pseudoscalar Nambu-Goldstone field. The scalar mass is then given by

$$m_\sigma^2=Z_H^{-1}\left[\frac{\sigma^2+\pi^2}{m_0^2}\right]^{\eta/(2+\eta)}\partial_\sigma^2 V(\sigma,\pi)\bigg|_{\substack{\sigma=m_0\\ \pi=0}}\ , \tag{2.10}$$

which depends on the common wave function renormalization constant $Z_H^{1/2}$ for the σ,π fields.

The extraction of Z_H is accomplished by evaluating the $A_\mu-\pi$ two-point function exploiting the two different couplings and retaining terms up to linear in the external momentum. Using the softly broken axial U(1) Ward identity to fix the form of the pseudoscalar vertex as

$$\Gamma_P^0(p+k,p)=\frac{\Sigma(p)}{m_0}+k^\mu p_\mu\frac{\partial_{p^2}\Sigma(p)}{m_0}\cdots\ , \tag{2.11}$$

a straightforward calculation [16] gives

$$Z_H=\frac{1}{Z_P^2}\frac{F_\pi^2}{\Sigma_0^2}\,, \tag{2.12}$$

where $Z_P=m_0/\Sigma_0$ and

$$F_\pi^2=\frac{1}{8\pi^2}\int_0^{\Lambda^2}dp^2\frac{p^2\Sigma^2(p)}{[p^2+\Sigma^2(p)]^2}\times\left[1-\frac{p^2}{2}\partial_{p^2}\ln\Sigma(p)\right]. \tag{2.13}$$

Since $\Sigma(p)\sim p^{\sqrt{1-\alpha/\alpha_c}-1}$, it follows that F_π is finite in the continuum limit $(\Lambda/\Sigma_0\to\infty)$ for all $0<\alpha\le\alpha_c$. However, since the integral receives nontrivial contributions from all momentum scales, its explicit evaluation requires knowledge of $\Sigma(p)$ for all p. A rough order-of-magnitude estimate gives F_π/Σ_0 to be a number of order unity for any $\alpha>0$. On the other hand, at $\alpha=0$, F_π^2 is dominated by the ultraviolet and goes as $\ln(\Lambda/\Sigma_0)$; which is the well established result of the pure NJL model.

The magnitude of the scalar mass depends on the strength of the four-fermion coupling and its deviation from criticality. Fine-tuning to the critical scaling region so that

$$\frac{1}{G_0}-\frac{\Lambda^2}{\pi^2}\frac{1}{(1+\omega)^2}\simeq\Lambda^2(\Sigma_0/\Lambda)^{2\omega},$$

and, using the fact that $Z_P\to(\Sigma_0/\Lambda)^{1-\omega}$, it follows that $m_\sigma^2\to\Sigma_0^2$. Thus, in the scaling region, the scalar mass is comparable to the fermion mass scale. This is consistent with calculation of the zero-momentum renormalized scalar denominator function given in Eq. (1.6). As for the pseudoscalar state, since

$$\partial_\pi^2V(\sigma,\pi)|_{\sigma=m_0,\,\pi=0}-0\,,$$

it follows that $m_\pi^2=0$ as is necessary for the Nambu-Goldstone boson of spontaneously broken chiral symmetry.

Before closing this section, we consider the very weak gauge coupling limit $\alpha\to0$. From an examination of the effective potential of Eq. (2.7), we immediately see that some modifications are necessary in this limit as evidenced by the factor of α^{-1} appearing in the last term. This dilemma can be readily alleviated by including additional terms in the asymptotic solution to the Schrödinger-Dyson equation which are subdominant except in the $\alpha\to0$ limit.

Including the next leading term, the asymptotic solution for Σ in the range $0\le\alpha<\alpha_c$ is given by

$$\Sigma(p)\sim\frac{\tilde{A}}{2\omega}e^{\omega\delta}\Sigma_0\left[\frac{\Sigma_0}{p}\right]^{1-\omega}-\frac{\tilde{A}}{2\omega}e^{-\omega\delta}\frac{\Sigma_0^3}{p^2}\left[\frac{\Sigma_0}{p}\right]^{-1+\omega}+\left[\frac{\tilde{A}e^{\omega\delta}}{2\omega}\right]^3\frac{1+\omega}{8(2-\omega)}\frac{\Sigma_0^3}{p^2}\left[\frac{\Sigma_0}{p}\right]^{3(1-\omega)}. \tag{2.14}$$

For any $\alpha>0$, the last term can be neglected for $p\gg\Sigma_0$ and we regain the previous solution, Eq. (1.2a). On the other hand, to make the connection with the $\alpha=0$ NJL model, this term needs to be retained since the Σ_0/p dependence of the last two terms is identical as $\alpha\to0$. The various terms in Eq. (2.14) correspond to the contributions from the operators σ, $\bar{\psi}\psi$, and σ^3, respectively in the operator-product expansion for the fermion propagator. This identification is consistent with the fact that σ and $\bar{\psi}\psi$ carry the physical mass dimensions $d_\sigma=2-\omega$ and $d_{\bar{\psi}\psi}=2+\omega$. On comparison with the $O(\alpha)$ perturbative evaluation of the self-energy for a fermion of mass Σ_0,

$$\Sigma(p)=\Sigma_0\left[1+\frac{3\alpha}{4\pi}+\frac{3\alpha}{2\pi}\ln\frac{\Sigma_0}{p}+\cdots\right]+\frac{\Sigma_0^3}{p^2}\left[-\frac{3\alpha}{4\pi}+\frac{3\alpha}{2\pi}\ln\frac{\Sigma_0}{p}+\cdots\right], \tag{2.15}$$

we identify

$$\tilde{A}=1+\frac{3}{8}\frac{\alpha}{\pi}+\cdots\,,\qquad e^{\omega\delta}=2\left[1-\frac{9}{8}\frac{\alpha}{\pi}+\cdots\right]. \tag{2.16}$$

Using the asymptotic solution, we compute the fermion condensate and bare mass for $\alpha\ll1$ as

$$\langle\bar{\psi}\psi\rangle\underset{\alpha\ll1}{\to}\frac{\Lambda^3}{3\pi}\frac{1}{\alpha}\left[-3\frac{\alpha}{2\pi}\left[\frac{\Sigma_0}{\Lambda}\right]^{2-\omega}+\frac{1}{2}\left[1+\frac{9\alpha}{4\pi}\right]\left[\frac{\Sigma_0}{\Lambda}\right]^{2+\omega}-\frac{1}{2}\left[1+\frac{9\alpha}{4\pi}\right]\left[\frac{\Sigma_0}{\Lambda}\right]^{6-3\omega}\right] \tag{2.17}$$

and

$$m_0\underset{\alpha\ll1}{\to}\frac{\Lambda}{2}\left[2\left[\frac{\Sigma_0}{\Lambda}\right]^{2-\omega}-\frac{3\alpha}{8\pi}\left[\frac{\Sigma_0}{\Lambda}\right]^{2+\omega}-\frac{9\alpha}{8\pi}\left[\frac{\Sigma_0}{\Lambda}\right]^{6-3\omega}\right]. \tag{2.18}$$

The effective potential in this limit can be obtained in the same manner as previously, yielding

$$V(\sigma,\pi)\underset{\alpha\ll1}{\to}\frac{\mu_0^2}{2G_0}-\frac{\mu_0}{G_0}\sigma+\frac{1}{2}\left[\frac{1}{G_0}-\frac{\Lambda^2}{(2\pi)^2}\left[1+\frac{3\alpha}{2\pi}\right]\right](\sigma^2+\pi^2)+\frac{1}{12\pi}\frac{1}{\alpha}\left[1+\frac{9\alpha}{4\pi}\right]\left[\frac{2-\omega}{4}\right](\Lambda^2)^{2(1-\omega)/(2-\omega)}(\sigma^2+\pi^2)^{2/(2-\omega)}-\frac{1}{12\pi}\frac{1}{\alpha}\left[1+\frac{9\alpha}{4\pi}\right]\frac{1}{4}(\sigma^2+\pi^2)^2+\cdots. \tag{2.19}$$

Note that the singular α^{-1} pieces in the last two terms identically cancel in the $\alpha \to 0$ limit, leaving the NJL result

$$V_{\mathrm{NJL}}(\sigma,\pi)=\frac{\mu_0^2}{2G_0}-\frac{\mu_0}{G_0}\sigma+\frac{1}{2}\left[\frac{1}{G_0}-\frac{\Lambda^2}{(2\pi)^2}\right](\sigma^2+\pi^2)+\frac{1}{16\pi^2}(\sigma^2+\pi^2)^2\ln\left[\frac{\Lambda^2}{\sigma^2+\pi^2}\right]+\frac{1}{32\pi^2}(\sigma^2+\pi^2)^2 . \tag{2.20}$$

Minimizing the effective potential of Eq. (2.20) yields the equation of state in the very weak gauge coupling limit:

$$\frac{\mu_0}{G_0}=\left[\frac{1}{G_0}-\frac{\Lambda^2}{(2\pi)^2}\left[1+\frac{3\alpha}{2\pi}\right]\right]m_0+\frac{1}{12\pi}\frac{1}{\alpha}\left[1+\frac{9\alpha}{4\pi}\right](\Lambda^2)^{2(1-\omega)/(2-\omega)}m_0^{(2+\omega)/(2-\omega)}-\frac{1}{12\pi}\frac{1}{\alpha}\left[1+\frac{9\alpha}{4\pi}\right]m_0^3 , \tag{2.21}$$

where m_0 is related to $\langle\bar{\psi}\psi\rangle$ through Eq. (1.3). Once again the m_0^3 term is required to secure a nonsingular $\alpha \to 0$ limit.

This term could also play a non-negligible role for small but nontrivial α when larger values of μ_0 and/or the condensate appear. As such, the data from the lattice simulations [14,17] for quenched QED should be accurately described by the parametrization

$$\mu_0=A\langle\bar{\psi}\psi\rangle+B(\mu_0+D\langle\bar{\psi}\psi\rangle)^{\delta}+C(\mu_0+D\langle\bar{\psi}\psi\rangle)^3 . \tag{2.22}$$

Here A, B, C, D, and δ are smooth nonuniversal functions of the coupling constants to be fitted. The normal fitting procedure would require that Eq. (2.22) be numerically inverted to express $\langle\bar{\psi}\psi\rangle$ as a function of the parameters A, B, C, D, δ, and μ_0. This fitting procedure is further complicated by the fact that, for smaller values of α, the exponent δ should be close to 3, so there will be large cancellations between the B and C terms. Consequently, their respective contributions will be hard to separate and the extraction of the exponent is difficult. This, in turn, may require the simulations to be performed using smaller bare masses. On the other hand, for larger α values, the various individual contributions should be more clearly identifiable and the fit more easily implementable.

III. EFFECTIVE DYNAMICS AT THE CRITICAL END POINT: $\alpha \to \alpha_c$, $G \to 1$

At the strong-gauge-coupling end point of the critical line where $\alpha \to \alpha_c$ and $G \to 1$, the four-fermion operators carry mass scaling dimension 4 and thus become marginal. Consequently, the physics in the vicinity of this point can be substantially different than for $\alpha < \alpha_c$. To construct the effective potential in this limit, we must solve the parametric equations [cf. Eqs. (1.4), (1.5), and (2.4)]

$$\partial_{m_0}W(m_0,0)=\langle\bar{\psi}\psi\rangle=-\frac{\tilde{A}}{2\pi^2}\Sigma_0^2\Lambda\left[\ln\left[\frac{\Lambda}{\Sigma_0}\right]+\delta-1\right] , \tag{3.1}$$

$$m_0=\frac{\tilde{A}}{2}\frac{\Sigma_0^2}{\Lambda}\left[\ln\left[\frac{\Lambda}{\Sigma_0}\right]+\delta+1\right] . \tag{3.2}$$

Because of the logarithmic factors arising in the critical coupling ($\alpha \to \alpha_c$) solution of the ladder Schwinger-Dyson equation, these equations cannot be analytically inverted in closed form. Nonetheless, the integration can still be performed giving

$$W(\sigma,\pi)=-\frac{\Lambda^2(\sigma^2+\pi^2)}{2\pi^2}\left\{1-\frac{2}{\ln(\Lambda/\Sigma_0)+\delta+1}+\frac{1}{2}\frac{1}{[\ln(\Lambda/\Sigma_0)+\delta+1]^2}\right\} . \tag{3.3}$$

Here $\Sigma_0=\Sigma_0(\sigma^2+\pi^2)$ is given implicitly through Eq. (3.2) where m_0^2 is replaced by $\sigma^2+\pi^2$. Using Eq. (2.3), the full effective potential then takes the form

$$V(\sigma,\pi)=\frac{\mu_0^2}{2G_0}-\frac{\mu_0}{G_0}\sigma+\frac{1}{2}\left[\frac{1}{G_0}-\frac{\Lambda^2}{\pi^2}\right](\sigma^2+\pi^2)+\frac{\Lambda^2}{\pi^2}(\sigma^2+\pi^2)\left\{\frac{1}{\ln(\Lambda/\Sigma_0)+\delta+1}-\frac{1}{4}\frac{1}{[\ln(\Lambda/\Sigma_0)+\delta+1]^2}\right\} . \tag{3.4}$$

Once again, the first two terms are μ_0 dependent and provide a soft explicit chiral and scale symmetry breaking. Now, however, the effective-mass term is scale invariant since the σ,π fields are mass dimension 2 at $\alpha=\alpha_c$. Finally the last term is approximately scale invariant with the symmetry breaking coming from the logarithm factors in the denominator.

Using the effective potential, we can also exact the critical scaling law giving the dependence of the order parameter $\langle\bar{\psi}\psi\rangle$ on the explicit symmetry-breaking parameter μ_0. Minimizing the potential at the critical end point where $1/G_0=\Lambda^2/\pi^2$ gives

$$\Sigma_0^2|_{\mathrm{critical}}=\frac{\mu_0\Lambda}{\tilde{A}} . \tag{3.5}$$

It then follows that

$$\langle\bar{\psi}\psi\rangle|_{\mathrm{critical}}=-\frac{\Lambda^2\mu_0}{(2\pi)^2}\left[\ln\left[\frac{\Lambda\tilde{A}}{\mu_0}\right]+2\delta-2\right] \tag{3.6}$$

which constitutes the critical scaling law for the $\alpha \to \alpha_c$, $G \to 1$ strong-coupling end point.

In the chiral limit ($\mu_0=0$), the effective potential is minimized for

$$\Sigma_0=\Lambda e^{\delta+1}\exp\left[\frac{-2\Lambda^2}{\pi^2(\Lambda^2/\pi^2-1/G_0)}\right]. \tag{3.7}$$

Thus when G_0 is tuned to be near criticality so that $\Lambda^2/\pi^2-1/G_0 \ll \Lambda^2$, then an exponentially huge hierarchy of mass scales, $\Sigma_0 \ll \Lambda$, emerges. This reflects the flatness of the potential $V(\sigma,0)$ which is also apparent using an approximate form obtained by replacing $\ln(\Lambda/\Sigma_0)$ by $\frac{1}{4}[\Lambda^2/(\sigma^2+\pi^2)]$ so that, in the chiral limit,

$$V(\sigma,\pi)\approx\frac{1}{2}\left[\frac{1}{G_0}-\frac{\Lambda^2}{\pi^2}\right](\sigma^2+\pi^2)+\frac{4\Lambda^2}{\pi^2}(\sigma^2+\pi^2)\frac{1}{\ln\left[\frac{\Lambda^2}{\sigma^2+\pi^2}\right]}. \tag{3.8}$$

A flat direction in the potential also often signals an abnormally light excitation in the spectrum. The mass of the composite σ state, however, is given by [cf. Eq. (2.10)]

$$m_\sigma^2 \simeq Z_H^{-1}\left[\frac{\sigma^2+\pi^2}{m_0^2}\right]^{\eta/(2+\eta)}\partial_\sigma^2 V(\sigma,\pi)\Big|_{\substack{\sigma=m_0\\ \pi=0}}, \tag{3.9}$$

where

$$Z_H=\frac{1}{Z_P^2}\frac{F_\pi^2}{\Sigma_0^2}.$$

Using Eq. (3.2) in its definition Eq. (1.7a), we find that for $\alpha\to\alpha_c$,

$$Z_P=\frac{\tilde{A}}{2}\frac{\Sigma_0}{\Lambda}\left[\ln\left[\frac{\Lambda}{\Sigma_0}\right]+\delta+1\right],$$

and hence

$$Z_H^{-1}=\frac{\tilde{A}^2\Sigma_0^4}{4\Lambda^2F_\pi^2}\left[\ln\left[\frac{\Lambda}{\Sigma}\right]+\delta+1\right]^2. \tag{3.10}$$

It follows that $m_\sigma\approx(\tilde{A}/2\pi)(\Sigma_0/F_\pi)\Sigma_0$ where F_π is given in Eq. (2.13). Since

$$\Sigma(p)\underset{\alpha\to\alpha_c}{\sim}\frac{1}{p}\ln(p),$$

F_π and consequently m_σ are again of order the dynamically generated fermion mass scale Σ_0 and not abnormally light. This is the case even though the potential is flat along the σ direction.

The above argument, used to estimate the mass of the composite scalar state for the strong-gauge-coupling end-point solution, is somewhat indirect. In particular, we never directly produce a kinetic-energy term for this degree of freedom and thus far fail to explicitly establish that it does in fact propagate. It would clearly be desirable to demonstrate the existence of a massive bound-state pole in the scalar channel of the fermion-antifermion scattering amplitude. Such a massive scalar bound-state pole would appear as a zero in the renormalized scalar denominator function

$$D_S^R(k^2)=Z_S^2\left[\frac{1}{G_0}+B_S^0(k^2)\right], \tag{3.11}$$

with $Z_S=\partial_{\Sigma_0}m_0$. Consequently we need to be able to estimate this function. Towards this end, we conjecture a form for the bare scalar bubble function $B_S^0(k^2)$ as $\alpha\to\alpha_c$ and $G\to1$ using a dispersion relation based on some reasonable assumptions.

Since the fermion mass is roughly given by Σ_0, we assume that the threshold constituting the lower limit of the dispersion integral occurs at $4\Sigma_0^2$, while for the upper limit we use the ultraviolet cutoff Λ^2. Next, we assume that the k^2 dependence of the numerator should be no higher than that obtained in free field theory. Thus we introduce two spectral functions, $\rho_1(k^2),\rho_2(k^2)$, and express the bubble sum as

$$B_S^0(k^2)=\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s-k^2}\rho_1(s)+k^2\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s-k^2}\frac{\rho_2(s)}{s}. \tag{3.12}$$

Finally, motivated by the approximate scale invariance, as reflected by the effective potential, Eq. (3.4), we further assume that the absorptive part of the spectral functions have no Σ_0^2 dependence above threshold. That is, the only Σ_0^2 dependence occurs in the lower limit of the dispersion integrals. Using the explicit functional form at $k^2=0$ given by [6]

$$\begin{aligned}B_S^{(0)}&=\partial_{m_0}\langle\bar\psi\psi\rangle\\&=-\frac{\Lambda^2}{\pi^2}+\frac{4\Lambda^2}{\pi^2}\frac{1}{\ln(\Lambda^2/\Sigma_0^2)+2\delta+1}\\&=\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s}\rho_1(s),\end{aligned} \tag{3.13}$$

it follows that

$$\begin{aligned}-\Sigma_0^2\partial_{\Sigma_0^2}B_S^0(0)&=\rho_1(4\Sigma_0^2)\\&=-\frac{4\Lambda^2}{\pi^2}\frac{1}{[\ln(\Lambda^2/\Sigma_0^2)+2\delta+1]^2}.\end{aligned} \tag{3.14}$$

Since Σ_0^2 can be varied arbitrarily we find

$$\rho_1(s)=-\frac{4\Lambda^2}{\pi^2}\frac{1}{[\ln(4\Lambda^2/s)+2\delta+1]^2}. \tag{3.15}$$

Thus the s dependence of the $\rho_1(s)$ is tracked from the known Σ_0^2 dependence of $B_S^0(0)$ and Eq. (3.12) takes the form

$$\begin{aligned}B_S^0(k^2)=&-\frac{4\Lambda^2}{\pi^2}\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s-k^2}\frac{1}{[\ln(4\Lambda^2/s)+2\delta+1]^2}\\&+k^2\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s-k^2}\frac{\rho_2(s)}{s}.\end{aligned} \tag{3.16}$$

To access the validity of this conjecture, we evaluate the integral at $k^2=0$ giving

$$B_S^0(0)=-\frac{\Lambda^2}{\pi^2}\frac{4}{\ln 4+2\delta+1}+\frac{4\Lambda^2}{\pi^2}\frac{1}{\ln\left[\frac{\Lambda^2}{\Sigma_0^2}\right]+2\delta+1}\ . \tag{3.17}$$

On comparison with the exact result, Eq. (3.13), we see that the correct Σ_0^2 dependence has been reproduced, while the coefficient of Λ^2 in the first term is approximately regained. [Since [6] $\delta(\alpha_c)\approx 0.7$, the coefficient of $-\Lambda^2/\pi^2$ using the conjecture is 0.95 compared to the exact value of 1.] Although there is this slight mismatch in the first term, we shall proceed with the conjecture as given and subsequently modify it to guarantee the correct $B_S^0(0)$ behavior.

To estimate the low-k^2 dependence of the bubble function, we approximate the first integral in Eq. (3.16) by replacing the s dependence of the logarithm in the integrand by its lower limit $4\Sigma_0^2$, giving

$$B_S^0(k^2)\underset{\text{low } k^2}{\approx}-\frac{4\Lambda^2}{\pi^2}\frac{\ln(\Lambda^2/4\Sigma_0^2)}{[\ln(\Lambda^2/\Sigma_0^2)+2\delta+1]^2}-\frac{4\Lambda^2}{\pi^2}\frac{\ln\left[\frac{4\Sigma_0^2}{4\Sigma_0^2-k^2}\right]}{[\ln(\Lambda^2/\Sigma_0^2)+2\delta+1]^2}+k^2\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s-k^2}\frac{\rho_2(s)}{s}\ . \tag{3.18}$$

Since the last two terms on the right-hand side vanish at $k^2=0$, it follows that we can guarantee the correct $B_S^0(0)$ if we replace the first term by the exact expression for $B_S^0(0)$ as given by Eq. (3.13). So doing we have

$$B_S^0(k^2)\underset{\text{low } k^2}{\approx}-\frac{\Lambda^2}{\pi^2}+\frac{4\Lambda^2}{\pi^2}\frac{1}{\ln(\Lambda^2/\Sigma_0^2)+2\delta+1}-\frac{4\Lambda^2}{\pi^2}\frac{\ln\left[\frac{4\Sigma_0^2}{4\Sigma_0^2-k^2}\right]}{[\ln(\Lambda^2/\Sigma_0^2)+2\delta+1]^2}+k^2\int_{4\Sigma_0^2}^{\Lambda^2}\frac{ds}{s-k^2}\frac{\rho_2(s)}{s}\ . \tag{3.19}$$

To proceed further, we need to estimate the ρ_2 spectral function. Unfortunately, this proves far more difficult. First of all, we can combine the assumed scale invariance of $\rho_2(s)$ above threshold with the known asymptotic behavior and write

$$\rho_2(s)=c\frac{4}{\pi^2}\frac{\Lambda^2}{[\ln(4\Lambda^2/s^2)+2\delta+1]^2}\ , \tag{3.20}$$

where c is a constant which much be determined. It is at this point that we face a serious impasse. In particular, the evaluation of c requires some additional data such as the k^2 derivative of $B_S^0(k^2)$ at some specified k^2 value. At present, such information is lacking. One possibility might be to try and relate such a derivative to F_π^2 and use the Pagels-Stokar formula. Even if this can be done, it still is not completely satisfactory since F_π^2 is only known via the integral form Eq. (2.13) which receives nontrivial contributions from all momentum scales.

Substituting the form for $\rho_2(s)$ given in Eq. (3.20) into Eq. (3.19) and evaluating the integral for low k^2 by again replacing the s dependence of the logarithm by its lower limit $4\Sigma_0^2$, we find

$$B_S^0(k^2)\underset{\text{low } k^2}{\approx}-\frac{\Lambda^2}{\pi^2}+\frac{4\Lambda^2}{\pi^2}\frac{1}{\ln(\Lambda^2/\Sigma_0^2)+2\delta+1}-(1-c)\frac{4\Lambda^2}{\pi^2}\frac{\ln(4\Sigma_0^2/4\Sigma_0^2-k^2)}{[\ln(\Lambda^2/\Sigma_0^2)+2\delta+1]^2} \tag{3.21}$$

with c an unspecified number of order 1.

The renormalized scalar denominator function at momentum k then takes the form

$$D_S^R(k^2)\underset{\text{low } k^2}{\simeq}\frac{\tilde{A}^2}{4\pi^2}\Sigma_0^2\left[\ln\left[\frac{\Lambda^2}{\Sigma_0^2}\right]+2\delta+1\right]^2\left[\frac{1}{G}-1\right]+\frac{\tilde{A}^2}{\pi^2}\Sigma_0^2\left[\ln\left[\frac{\Lambda^2}{\Sigma_0^2}\right]+2\delta+1\right]-\frac{\tilde{A}^2}{\pi^2}(1-c)\Sigma_0^2\ln\left[\frac{4\Sigma_0^2}{4\Sigma_0^2-k^2}\right]\ . \tag{3.22}$$

Application of the gap equation in the chiral limit

$$\ln\left[\frac{\Lambda^2}{\Sigma_0^2}\right]+2\delta=2\left[\frac{G+1}{G-1}\right]\ , \tag{3.23}$$

then gives, as $G\to 1$,

$$D_S^R(k^2)\underset{\text{low } k^2}{\approx}\frac{\tilde{A}^2}{\pi^2}\Sigma_0^2\left[1+(1-c)\ln\left[1-\frac{k^2}{4\Sigma_0^2}\right]\right]\ . \tag{3.24}$$

This denominator function vanishes at

$$k^2=4\Sigma_0^2\left[1-\exp\left[\frac{-1}{1-c}\right]\right]\ , \tag{3.25}$$

while

$$\partial_{k^2}D_S^R(k^2)\big|_{k^2=4\Sigma_0^2[1-\exp(\frac{-1}{1-c})]}=-\frac{\tilde{A}^2}{\pi^2}\frac{1-c}{4}\exp\left[\frac{1}{1-c}\right]\ . \tag{3.26}$$

Thus, in the vicinity of the zero, it can be written as

$$D_S^R(k^2)\approx-\frac{\tilde{A}^2}{\pi^2}\frac{1-c}{4}\exp\left[\frac{1}{1-c}\right]\times\left\{k^2-4\Sigma_0^2\left[1-\exp\left[\frac{-1}{1-c}\right]\right]\right\}\ . \tag{3.27}$$

This simple zero of $D_S^R(k^2)$ corresponds to a simple pole in the scalar channel of the fermion-antifermion scattering amplitude and hence a composite propagating scalar bound state of mass

$$m_\sigma = 2\left[1-\exp\left[\frac{-1}{1-c}\right]\right]^{1/2}\Sigma_0 .$$

One shortcoming in the above analysis is that we have completely neglected the effects of Coulomb bound states in the scalar channel. For small values of α, when the dominant scalar channel binding arises from the four-fermion interaction, the effects of these Coulomb bound states should not be very significant. As one approaches the critical end point, however, the binding due to the electromagnetic gauge interactions becomes comparatively important and the Coulomb bound states could play a more prominent role.

We reiterate that the above value for m_σ is based on using a conjectured dispersion relation which in turn involved certain unproven assumptions as well as depending on the unknown constant c. Thus, while the magnitude of the propagating scalar mass is found to be of the order of the fermion mass scale Σ_0 and hence consistent with other estimates, the specific numerical value is not yet fixed. A more precise determination will require still further study. If we make the additional, very strong assumption that the bubble function actually satisfies an unsubtracted dispersion relation, then the spectral function ρ_2 vanishes and hence $c=0$. However, we know from the known asymptotic behavior that, for weak α values, a subtraction is necessary. Thus we anticipate that this should also be the case at the critical end point. Assuming an unsubtracted dispersion relation is tantamount to being able to neglect all external momentum factors in the numerator of the bubble sum integral even after Feynman parametrization. This is clearly an extremely strong restriction which is most likely invalid. We further note that the assumption used to construct the dispersion relation at the strong-coupling end point cease to be valid for smaller α values where the four-fermion operators scale with mass dimension greater than 4. In that case, for instance, the s dependence of the spectral weights will no longer simply track the Σ_0^2 dependence of $B_S^0(0)$. This is born out by the explicit calculation in the pure NJL limit ($\alpha \to 0$).

IV. CONCLUSIONS

It is most natural to define the quantum version of a particular theory so as to preserve as many symmetries of the classical model as is possible. This often entails the introduction of additional operators in the Lagrangian than those appearing at the classical level. Quenched, planar QED in the chiral limit exhibits both chiral and scale symmetries in the classical limit. We have seen that, when chirally invariant four-fermion operators are included in addition to the gauge interactions, along the critical line separating the symmetric phase from that of spontaneously broken chiral symmetry, the scale symmetry is also preserved in the Wigner-Weyl realization. It follows that four-fermion operators should be included in the quantum description of ladder QED.

In fact, for $0<\alpha<\alpha_c$, the four-fermion interactions play a vital role in driving the theory to criticality. At first glance, this might be somewhat surprising since these operators are formally irrelevant and thus one might anticipate that their inclusion should lead to effects suppressed by inverse powers of the cutoff. However, it takes but a small amount of four-fermion coupling $[G_0 \sim (1/\Lambda^2)]$ to bring the theory to the vicinity of the critical line. Once in this critical scaling region, the composite scalar and pseudoscalar degrees of freedom become dynamically active and it is their relevant interactions which accurately describe the physics.

We have studied ladder QED in the vicinity of the entire critical line and the anomalous dimensions detailing the behavior in this region were extracted. Off the critical line, the scale symmetry is explicitly broken due to the dimensional four-fermion coupling. For weak to moderate gauge coupling and provided the four-fermion coupling is appropriately fine-tuned, the composite scalar mode becomes a propagating dynamical degree of freedom and its mass was estimated to be of the same order as the fundamental fermion mass.

At the strong-coupling end point ($\alpha \to \alpha_c$), the four-fermion operator becomes scale invariant in the continuum limit. Moreover, the effective potential is very flat along the σ direction. While this allows for a huge hierarchy of mass scales, $\Sigma_0 \ll \Lambda$, to emerge naturally, the composite scalar mass is still of the order of the dynamically generated fermion mass scale and no abnormally light dilaton emerges in the spectrum. Flat direction potentials appear in many diverse physical applications involving hierarchical mass scales and are usually accompanied by very light excitations. Here we have exhibited an example where the flat potential does not lead to such a nearly massless mode. Further investigations into the generalizations of such flat potentials and their possible implications for hierarchy models could prove very instructive.

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[1] K. Johnson, M. Baker, and R. Willey, Phys. Rev. **136**, B1111 (1964); **163**, 1699 (1967).
[2] S. L. Adler and W. A. Bardeen, Phys. Rev. D **4**, 3045 (1971).
[3] T. Maskawa and H. Nakajima, Prog. Theor. Phys. **52**, 1326 (1974); **54**, 860 (1975).
[4] R. Fukuda and T. Kudo, Nucl. Phys. **B117**, 250 (1976).
[5] V. A. Miransky, Nuovo Cimento **90A**, 149 (1985); Zh. Eksp. Teor. Fiz. **88**, 1514 (1985) [Sov. Phys. JETP **61**, 905 (1985)]; P. I. Fomin, V. P. Gusynin, V. A. Miransky, and Yu. A. Sitenko, Riv. Nuovo Cimento **6**, 1 (1983).
[6] C. N. Leung, S. T. Love, and W. A. Bardeen, Nucl. Phys. **B273**, 649 (1986); W. A. Bardeen, C. N. Leung, and S. T. Love, *ibid.* **B323**, 493 (1989).
[7] W. A. Bardeen, in *New Trends in Strong Coupling Gauge Theories,* Proceedings of the 1988 International Workshop, Nagoya, Japan, edited by M. Bando (World Scientific, Singapore, 1989), p. 110.
[8] B. Holdom, Phys. Lett. **150B**, 301 (1985); T. Appelquist, D. Karabali, and L. C. R. Wijewardhana, Phys. Rev. Lett. **57**, 957 (1986); T. Appelquist and L. C. R. Wijewardhana, Phys. Rev. D **35**, 774 (1987); **36**, 568 (1987); see also T. Appelquist, M. Einhorn, T. Takeuchi, and L. C. R. Wijewardhana, Phys. Lett. B **220**, 1043 (1989).
[9] W. A. Bardeen, C. N. Leung, and S. T. Love, Phys. Rev. Lett. **56**, 1230 (1986).
[10] W. A. Bardeen, S. T. Love, and V. A. Miransky, Phys. Rev. D **42**, 3514 (1990).
[11] V. P. Gusynin and V. A. Miransky, Mod. Phys. Lett. **6**, 2443 (1991).
[12] K.-I. Kondo, H. Mino, and K. Yamawaki, Phys. Rev. D **39**, 2430 (1989); T. Appelquist, M. Soldate, T. Takeuchi, and L. C. R. Wijewardhana, in *TeV Physics,* Proceedings of the 12th Johns Hopkins Workshop on Current Problems in Particle Theory, Baltimore, Maryland, 1988, edited by G. Domokos and S. Kovesi-Domokos (World Scientific, Singapore, 1989); W. A. Bardeen, C. N. Leung, and S. T. Love, Nucl. Phys. **B323**, 493 (1989); M. Inoue, T. Muta, and T. Ochiumi, Mod. Phys. Lett. A **4**, 605 (1989); K. I. Aoki, Kyoto University Report No. RIFP-758, 1988 (unpublished).
[13] Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961).
[14] A. Kocić, S. Hands, J. Kogut, and E. Dagotto, Nucl. Phys. **B347**, 217 (1990); E. Dagotto, J. Kogut, and A. Kocić, Phys. Rev. D **43**, R1763 (1991); J. B. Kogut, E. Dagotto, and A. Kocić, Phys. Rev. Lett. **60**, 772 (1988); Nucl. Phys. **B317**, 253 (1989); **B317**, 271 (1989); S. Hands, J. B. Kogut, and E. Dagotto, *ibid.* **B333**, 557 (1990).
[15] For an estimate of the scalar propagator in the symmetric phase at weak to moderate α values, see T. Appelquist, J. Terning, and L. C. R. Wijewardhana, Phys. Rev. D **44**, 871 (1991).
[16] S. L. Adler and W. A. Bardeen, Phys. Rev. D **4**, 3045 (1971); R. Jackiw and K. Johnson, *ibid.* **8**, 2386 (1973); H. Pagels and S. Stokar, *ibid.* **20**, 2947 (1979); J. M Cornwall, *ibid.* **22**, 1452 (1980).
[17] M. Gockeler, R. Horsley, E. Laermann, P. Rakow, G. Schierholz, R. Sommer, and U.-J. Wiese, Nucl. Phys. **B334**, 527 (1990).